



# Measurement of Light-Cone wave functions by diffractive dissociation

## Outline

Wave Functions, Structure Functions and Form Factors

Measurements of the LCWF and CT for pions. Reached Asymptotia?

Measurement of the EM component of the Photon LCWF

Measurement of the hadronic component of the Photon LCWF

LCWF of the proton

Summary

## Light-Cone Wave Functions:

Solutions of LC Hamiltonian:  $H_{LC}^{QCD}|\psi_h\rangle = M_h^2|\psi_h\rangle$

$$H_{LC}^{QCD} = P^+P^- - P_\perp^2$$

## Fock States Expansion for the Pion:

$$\begin{aligned} |\psi_{\pi^-}\rangle &= \sum_n \langle n | \pi^- \rangle |n\rangle \\ &= \psi_{d\bar{u}/\pi}^{(\Lambda)}(u_i, \vec{k}_{\perp i}, \lambda_i) |\bar{u}d\rangle \\ &\quad + \psi_{d\bar{u}g/\pi}^{(\Lambda)}(u_i, \vec{k}_{\perp i}, \lambda_i) |\bar{u}dg\rangle + \dots \end{aligned}$$

$$u_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{p^0 + p^z}, \quad \sum_{i=1}^n u_i = 1, \quad \sum_{i=1}^n \vec{k}_{\perp i} = \vec{0}_\perp.$$

$k_\perp$  is the quark (antiquark) transverse momentum.

ANY parton with fractional momentum  $x$ :

$$G_{a/h}(x, Q) = \sum_n \int d[\mu_n] \left| \Psi_{n/h}^{(Q)}(u_i, \vec{k}_{\perp i}, \lambda_i) \right|^2 \sum_i \delta(x - u_i)$$

Parton Distribution Functions Derived from *inclusive* DIS cross sections.

**VALENCE QUARK** with fractional momentum  $u$ :

$$\phi_{q\bar{q}}(u) \sim \int_0^{Q^2} \psi_{q\bar{q}}(u, k_{\perp}) dk_{\perp}^2 \quad Q^2 = \frac{k_{\perp}^2}{u(1-u)}$$

For not too small  $Q^2$ :

$$\psi(u, k_{\perp}) \sim \frac{\phi(u)}{k_{\perp}^2}$$

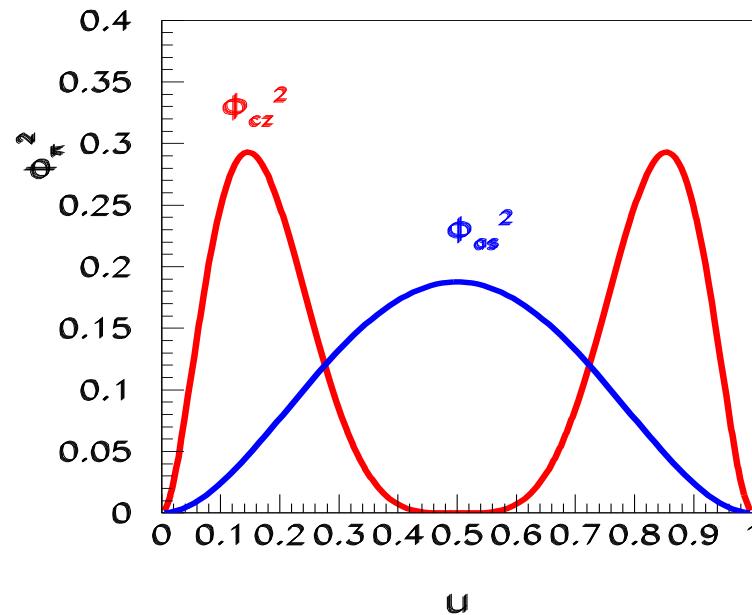
Distribution Amplitude  $\phi(u)$ , an *exclusive* quantity.

## PION DISTRIBUTION AMPLITUDES

$$\phi_\pi(u, \mu^2) = u(1-u) \sum_{n \geq 0} a_n C_n^{3/2} (2u-1) \left( \ln \frac{\mu^2}{\Lambda^2} \right)^{-\gamma_n/2\beta_2}$$

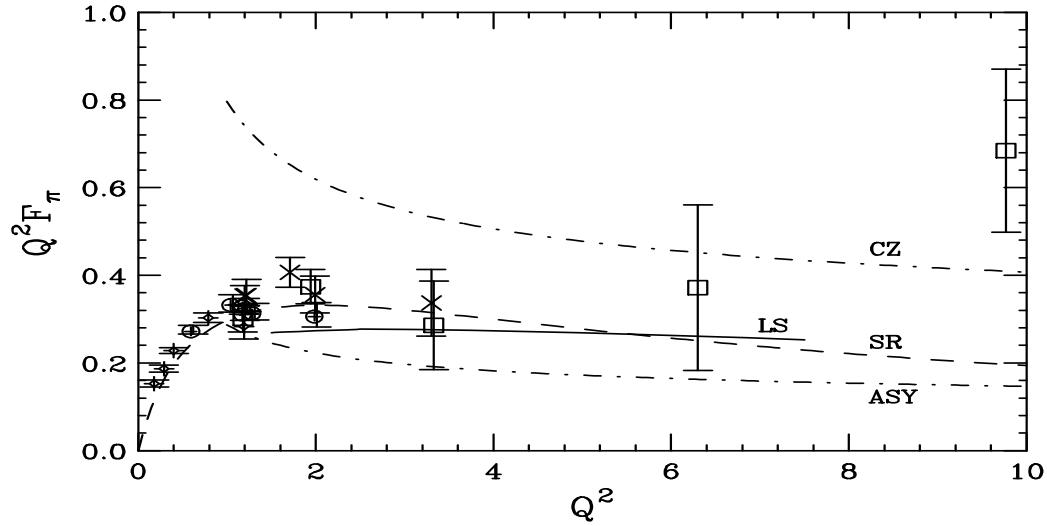
1. The Asymptotic Function:  $a_2 = 0, a_4 = 0$   
 pQCD; for  $Q^2 \rightarrow \infty$ :  $\phi_{Asy}(u) = \sqrt{3}f_\pi u(1-u)$

2. The Chernyak-Zhitnitsky Function:  $a_2 = 0.4, a_4 = 0$   
 QCD sum rules; for low  $Q^2$ :  $\phi_{cz}(u) = 5\sqrt{3}f_\pi u(1-u)(1-2u)^2$

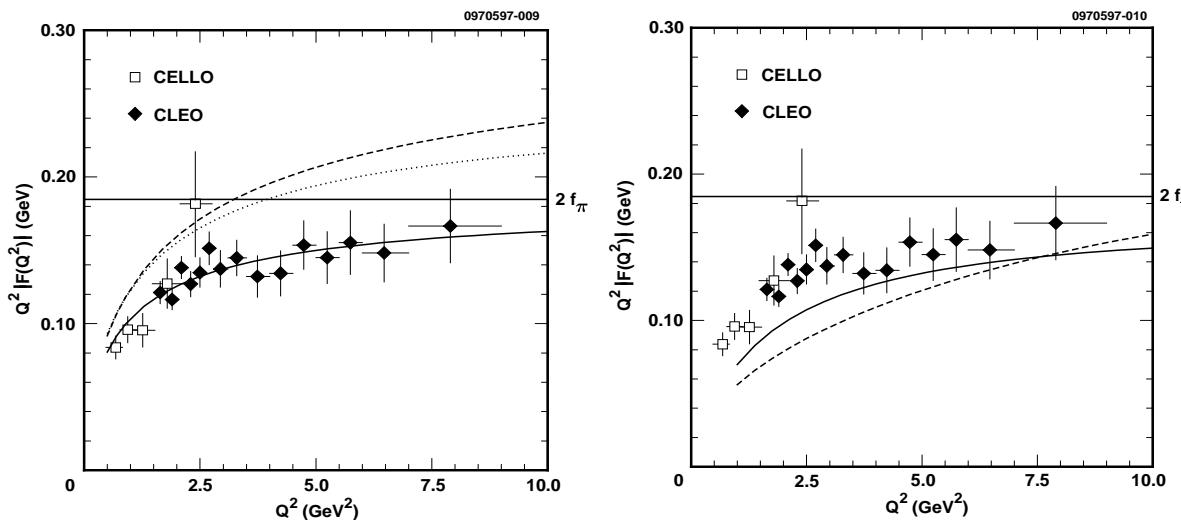


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- (1) G.P. Lepage, S.J. Brodsky, Phys. Lett.B87, 359 (1979)  
 A.V. Efremov, A.V. Radyushkin, Theor. Math. Phys. 42, 97 (1980).  
 (2) V.L. Chernyak and A.R. Zhitnitski, Phys. Rep. 112, 173 (1984).

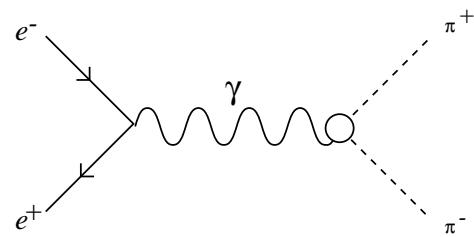
Space-Like  $\pi^+$  Form Factors (G. Sterman, P. Stoler, A.R.N.P.S. **43**, 193 (1997))



$\pi^0$  Transition Form Factor CLEO Col. Phys. Rev. D57, 33 (1998)

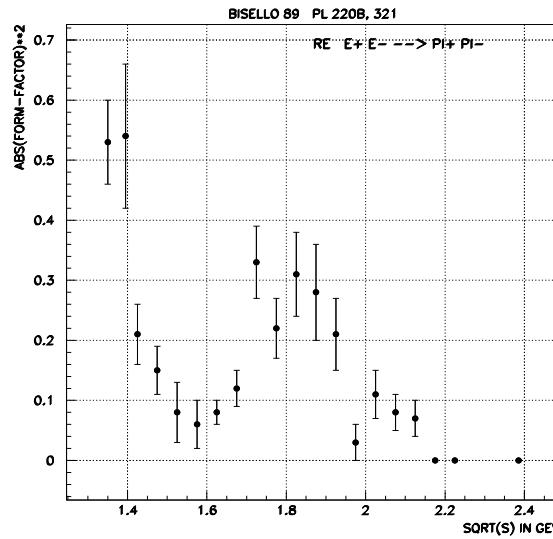


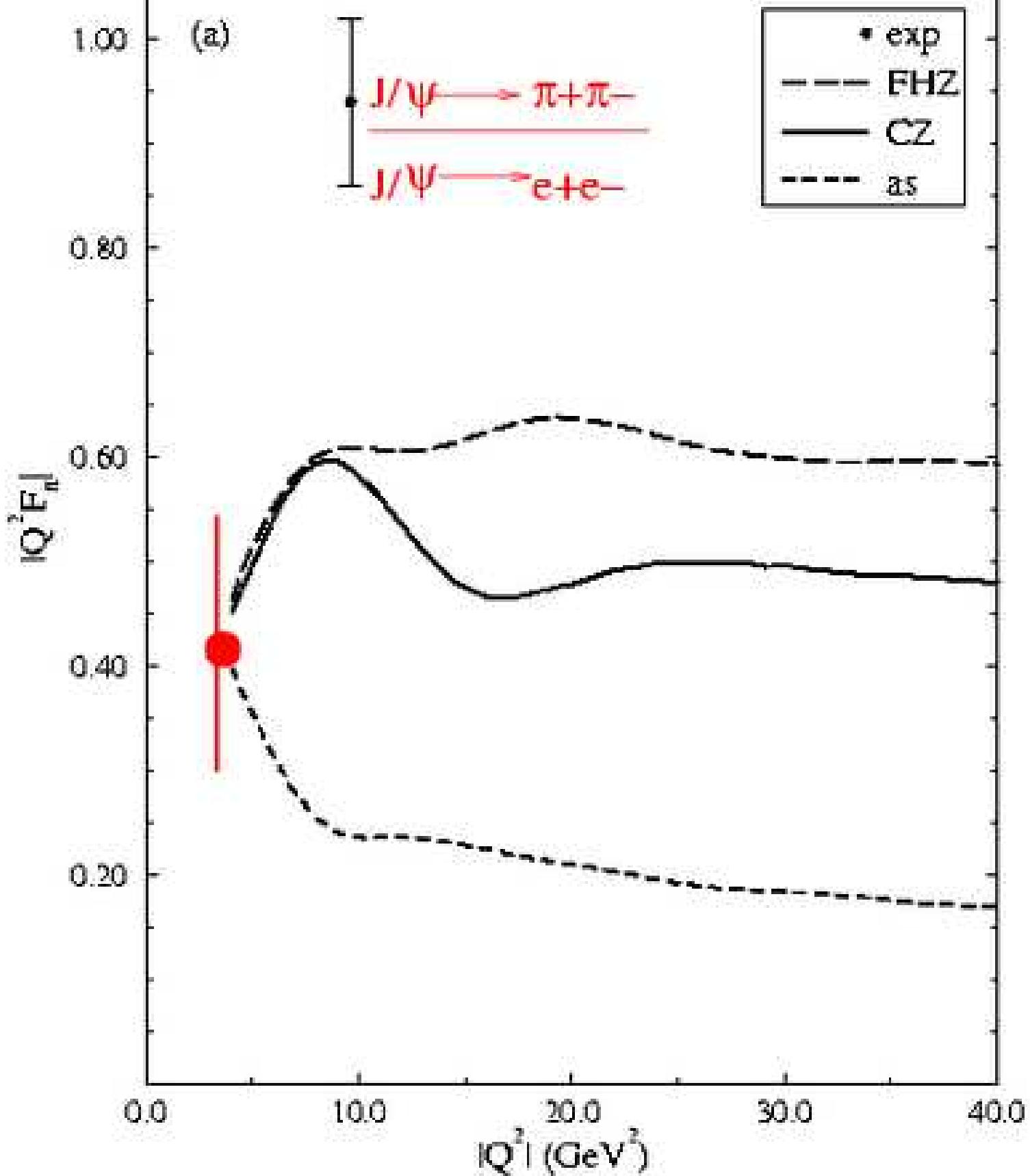
## Time-Like Form Factor ( $q^2 > 0$ )



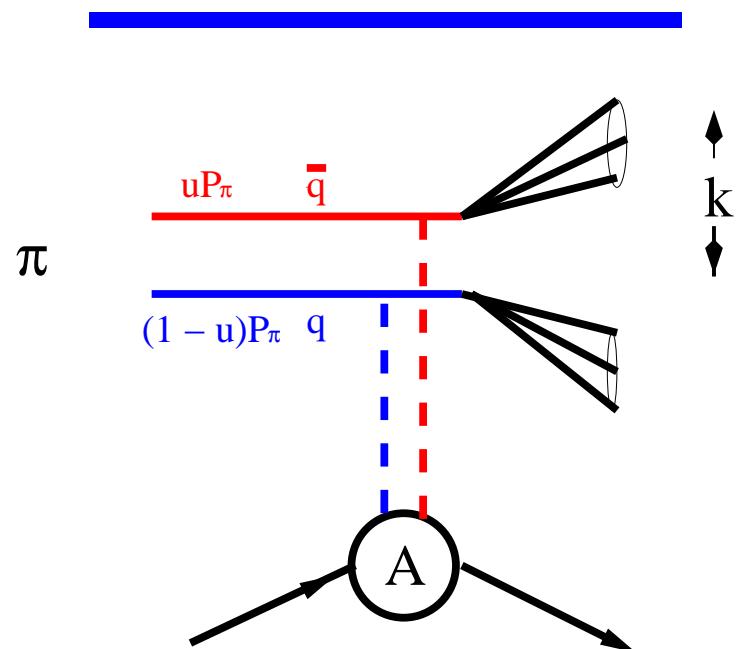
$$\sigma = \sigma(e^+ e^- \rightarrow q\bar{q}) |F_\pi(s)|^2 = \frac{\alpha^2 \pi}{3} \frac{(s - 4m_\pi^2)^{3/2}}{s^{5/2}} |F_\pi(s)|^2.$$

DM2 Coll. Phys. Lett. **B220**, 321 (1989)





## Differential Measurement of the Pion Wave Function



In the diffractive dissociation of the  $|q\bar{q} \rangle$  configuration into DJ,  $u$  can be measured by the momentum ratio of the two jets:

$$u_{measured} = \frac{p_{jet1}}{p_{jet1} + p_{jet2}}, \quad Q_{DJ}^2 = \frac{k_t^2}{u(1-u)}$$

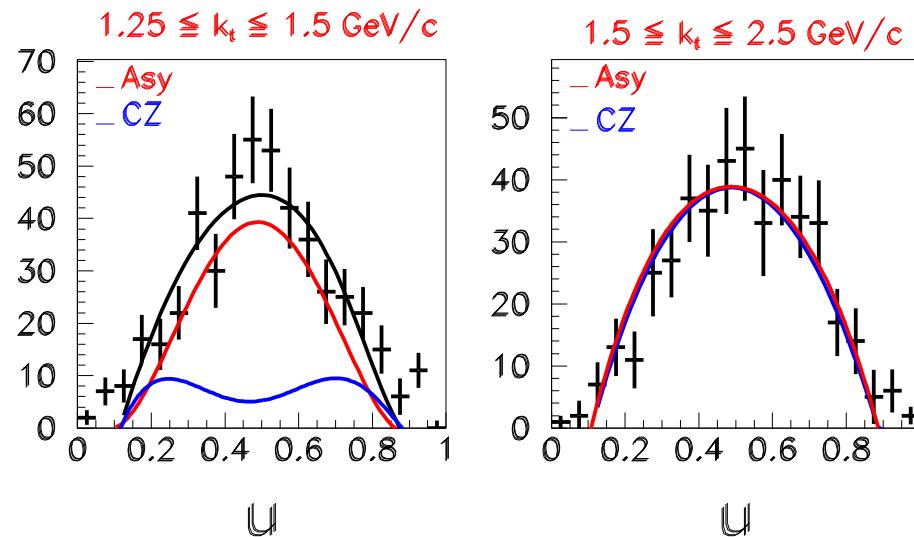
L. Frankfurt, G.A. Miller and M. Strikman,  
P.L. B304(1993)1

Cross-Section for Diffractive Dissociation to Dijet:

$$\frac{d^4\sigma_N}{dudM_J^2 \cdot d^2P_{N_t}} = 2.6 \text{ GeV}^{-6} \left(\frac{\text{GeV}}{\kappa_t}\right)^8 \phi^2(u)$$

## THE $q\bar{q}$ MOMENTUM WAVE FUNCTION MEASURED BY DI-JETS

Fermilab E791 Collaboration, PRL 86, 4768 (2001)



$1.5 \text{ GeV}/c \leq k_t \leq 2.5 \text{ GeV}/c; Q^2 \sim 16 (\text{GeV}/c)^2 :$        $\phi^2 > 0.9 \phi_{Asy}^2$

$1.25 \text{ GeV}/c \leq k_t \leq 1.5 \text{ GeV}/c; Q^2 \sim 8 (\text{GeV}/c)^2 :$

$\phi^2$  contains contributions from CZ or other non-perturbative wave functions

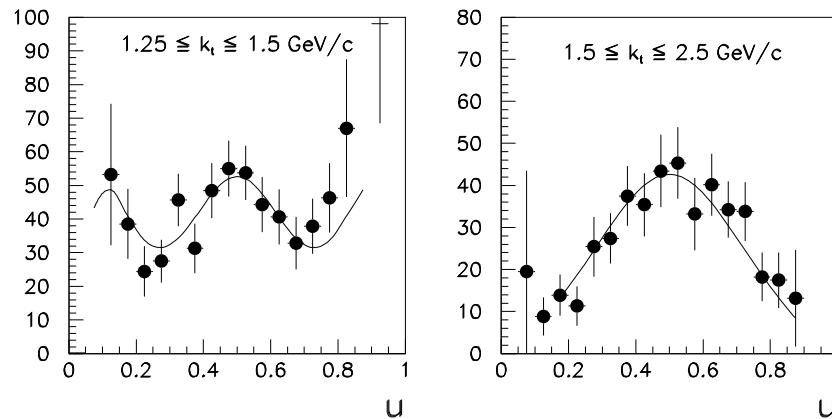
## Fit to Gegenbauer Polynomials

**Generate Acceptance-Corrected Momentum distributions**

**Assume  $\frac{d\sigma}{du} \propto \phi_\pi^2(u, Q^2)$  in both  $k_\perp$  regions**

**Fit distributions to:**

$$\frac{d\sigma}{du} \propto \phi_\pi^2(u, Q^2) = 36u^2(1-u)^2 \left(1.0 + a_2 C_2^{3/2}(2u-1) + a_4 C_4^{3/2}(2u-1)\right)^2$$



**For high  $k_t$  :  $a_2 = a_4 = 0 \rightarrow$  Asymptotic**

**For low  $k_t$  :  $a_2 = 0.30 \pm 0.05$ ,  $a_4 = (0.5 \pm 0.1) \cdot 10^{-2} \rightarrow$  Transition**

## THE $k_t$ DEPENDENCE OF DI-JETS YIELD

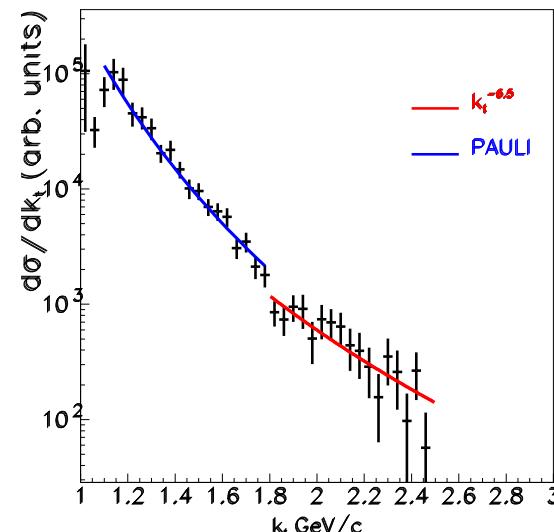
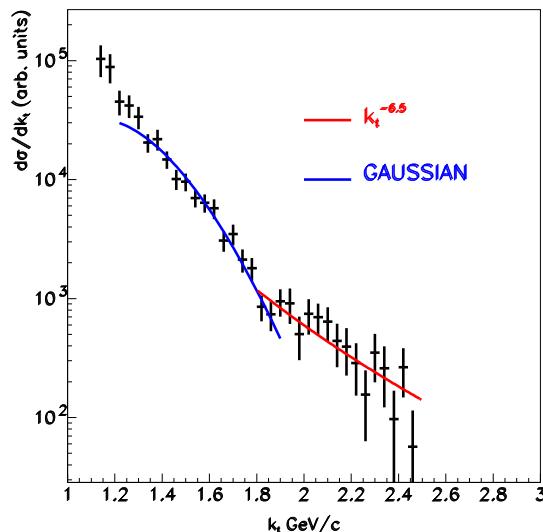
$$\frac{d\sigma}{dk_t^2} \propto |\alpha_s(k_t^2)G(x, k_t^2)|^2 \left| \frac{\partial^2}{\partial k_t^2} \psi(u, k_t) \right|^2$$

With  $\psi \sim \frac{\phi}{k_t^2}$ , weak  $\phi(k_t^2)$  and  $\alpha_s(k_t^2)$  dependences and  $G(x, k_t^2) \sim k_t^{1/2}$ :  $\frac{d\sigma}{dk_t} \sim k_t^{-6}$

For low  $k_t$ :

Gaussian:  $\psi \sim e^{-\beta k_t^2}$  (Jakob and Kroll)

Coulomb:  $\psi(p) = \left( \frac{1}{1+p^2/p_a^2} \right)^2$  (Pauli)



Is  $\sigma \propto \phi^2$  ??

hep-ph/0103275

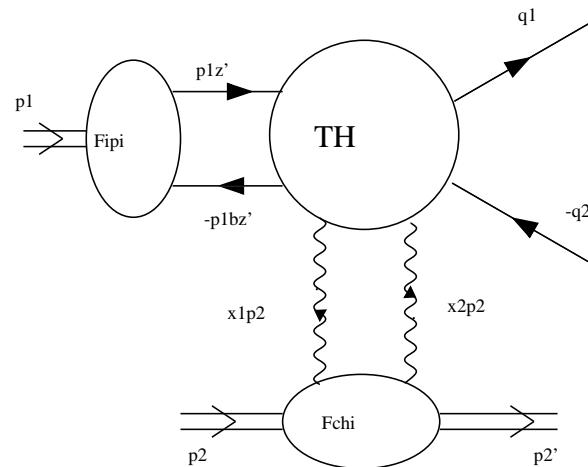
## QCD factorization for the pion diffractive dissociation to two jets

V.M. BRAUN<sup>1</sup>, D.YU. IVANOV<sup>1,2</sup> A. SCHÄFER<sup>1</sup> and L. SZYMANOWSKI<sup>1,3</sup>

hep-ph/0103295

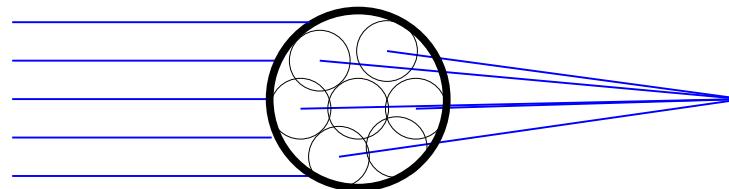
Has the E791 experiment measured  
the pion wave function profile ?

Victor Chernyak



When measured in a heavy nucleus with Color Transparency conditions  
 $\sigma \propto \phi^2$  (Brodsky, Frankfurt, Nikolaev, Braun...)

EXPECTED A-DEPENDENCE  
FOR COLOR TRANSPARENCY



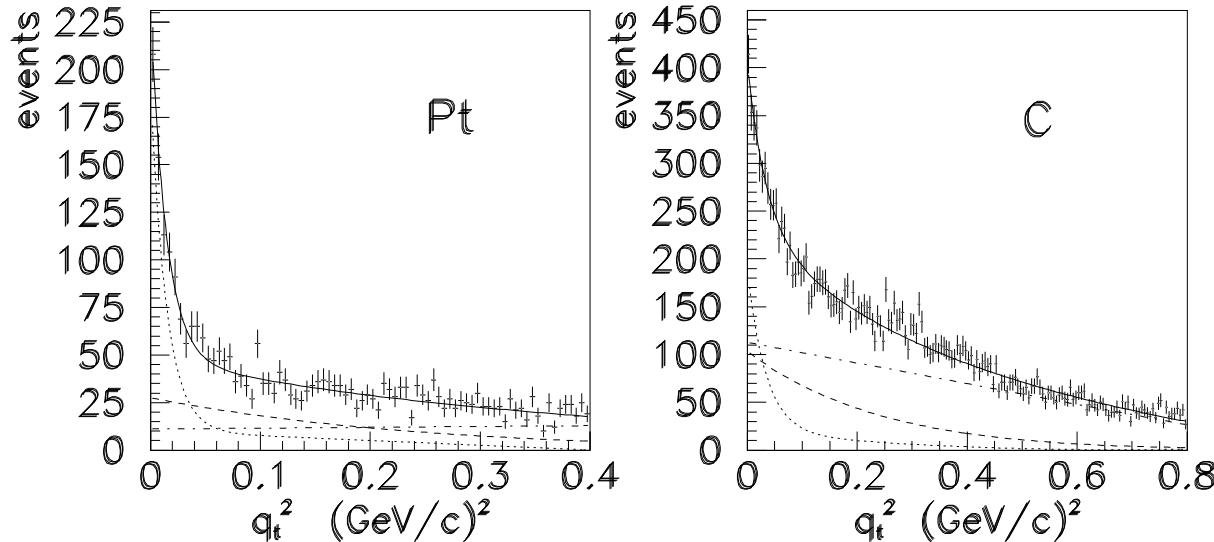
- Fully coherent interactions between pion and nucleons.
- Emerging Di-Jets do not interact with nucleus.

$$\mathcal{M}(A) = A \cdot \mathcal{M}(\mathcal{N})$$

$$\frac{d\sigma}{dq_t^2} \propto A^2 \quad q_t^2 \sim 0$$

$$\underline{\sigma \propto A^{4/3}}$$

E791 Collaboration, E. Aitala *et al.*, Phys. Rev. Lett. 86, 4773 (2001)



A-Dependence results:  $\sigma \propto A^\alpha$

<u><math>k_t</math> range (GeV/c)</u>	<u><math>\alpha</math></u>	<u><math>\alpha</math> (CT)</u>
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$1.25 < k_t < 1.5$	$1.64 +0.06 -0.12$	$1.25$
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$1.5 < k_t < 2.0$	$1.52 \pm 0.12$	$1.45$
-------------------	-----------------	--------

$2.0 < k_t < 2.5$	$1.55 \pm 0.16$	$1.60$
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$\alpha$  (Incoh.) =  $0.70 \pm 0.1$

# Measurements of the photon light-cone wave function

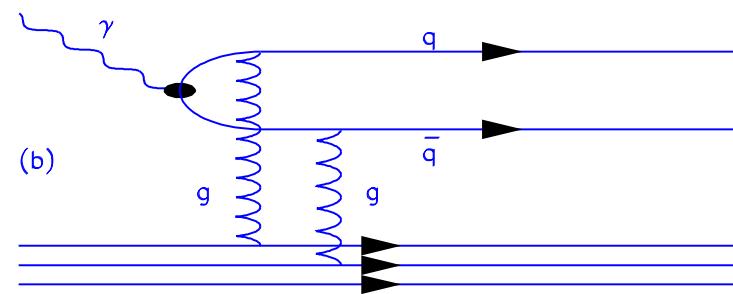
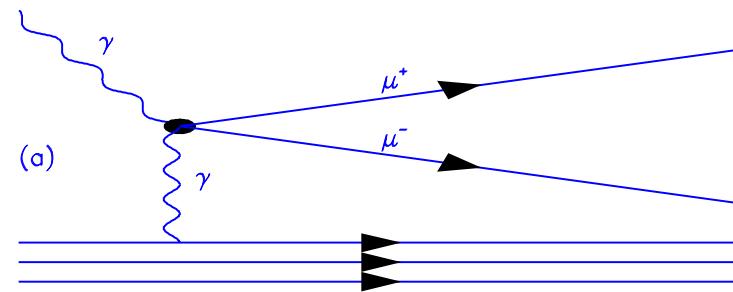
Fock state decomposition:

$$\begin{aligned}\psi_\gamma = & a|\gamma_p\rangle + b|l^+l^-\rangle + c|l^+l^-\gamma\rangle + (\text{other e.m.}) + d|q\bar{q}\rangle + e|q\bar{q}g\rangle \\ & + (\text{other had.}) + \dots\end{aligned}$$

Electromagnetic and Hadronic components

Real and Virtual Photons

Transverse and Longitudinal polarizations



## The electromagnetic $|l^+l^-\rangle$ component of the photon

The wave function of the first component of a photon with virtuality  $Q^2$  is<sup>1</sup>:

$$\psi_{\lambda_1\lambda_2}^\lambda(k_\perp, u) = -ee_l \frac{\bar{l}_{\lambda_1}(k)\lambda \cdot \epsilon^\lambda l_{\lambda_2}(q-k)}{\sqrt{u(1-u)} \left(Q^2 + \frac{k_\perp^2 + m^2}{u(1-u)}\right)}$$

The distribution amplitude (squared) for transversely polarized photons:

$$\Phi_{l\bar{l}/\gamma_T^*}^2(u, k_\perp) \sim \sum_{\mu=1}^2 \frac{1}{4} Tr \psi_{\gamma^*}^2 = \frac{m_l^2 + k_\perp^2 [u^2 + (1-u)^2]}{[k_\perp^2 + a_l^2]^2}, \quad a_l^2 = m_l^2 + Q^2 u(1-u).$$

For longitudinally polarized photons:

$$\Phi_{l\bar{l}/\gamma_L^*}^2 \sim \frac{Q^2 [u^2(1-u)^2]}{[k_\perp^2 + a^2]^2}$$

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(1) S. J. Brodsky, L. Frankfurt, J. F. Gunion, A. H. Mueller and M. Strikman, Phys. Rev. D50, 3134 (1994)

## The hadronic $|q\bar{q}\rangle$ component of the photon

For  $k_\perp^2 \gg \Lambda_{QCD}^2$  the wave function of the  $|q\bar{q}\rangle$  component of a photon with virtuality  $Q^2$  is the same as for the  $|l^+l^-\rangle$  component.

The distribution for massive quarks and transversely polarized photons:

$$\Phi_{q\bar{q}/\gamma_T^*}^2(u, k_\perp) \sim \frac{m_q^2 + k_\perp^2[u^2 + (1-u)^2]}{[k_\perp^2 + a_q^2]^2}$$

where:

$$a_q^2 = m_q^2 + Q^2 u(1-u).$$

For low  $k_\perp^2$  other predictions:

1. Instanton models  
(1999)

Petrov *et al.*, Phys.Rev. D59, 11401

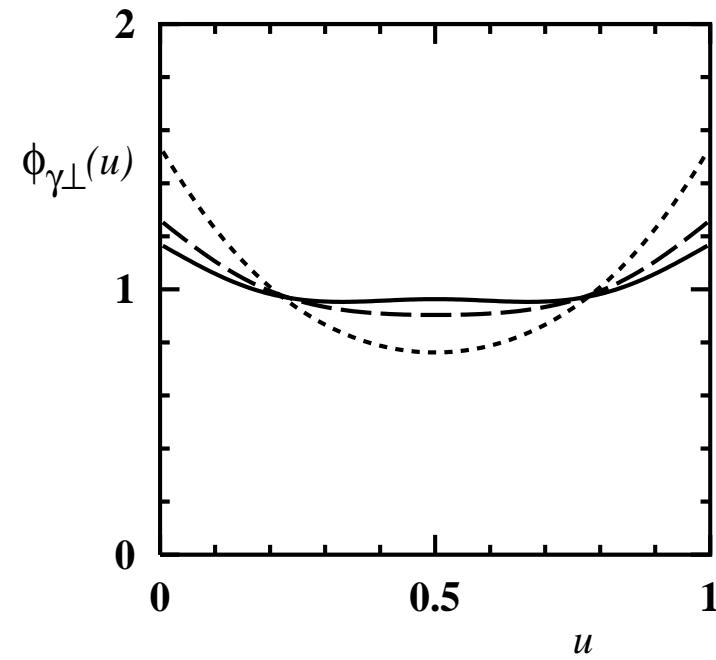
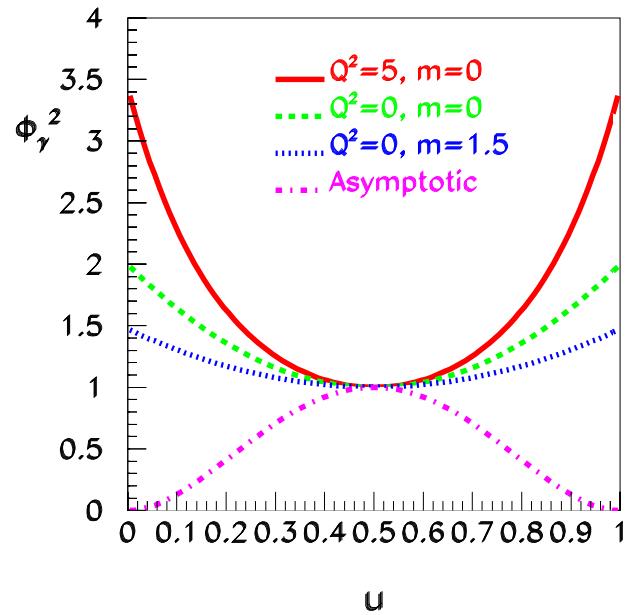
2. Asymptotic distribution:  $\Phi_{q\bar{q}/\gamma}^2 \sim u^2(1-u)^2$  Balitsky *et al.*, Nucl. Phys. B312, 509 (1989)

# $\Phi$ Distributions of $|l^+l^-\rangle$ , $|q\bar{q}\rangle$ components of the photon

Instanton Wave Functions, real photons (solid line), virtual

Perturbative Wave functions

$Q^2 = 250 \text{ MeV}^2$  (dashed line),  $Q^2 = 500 \text{ MeV}^2$  (dotted line).



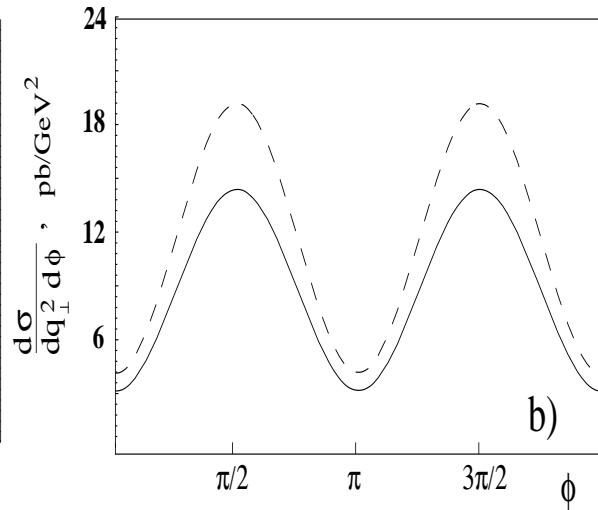
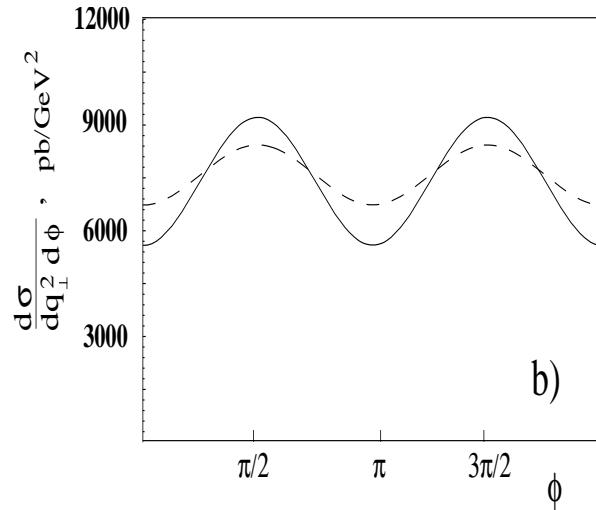
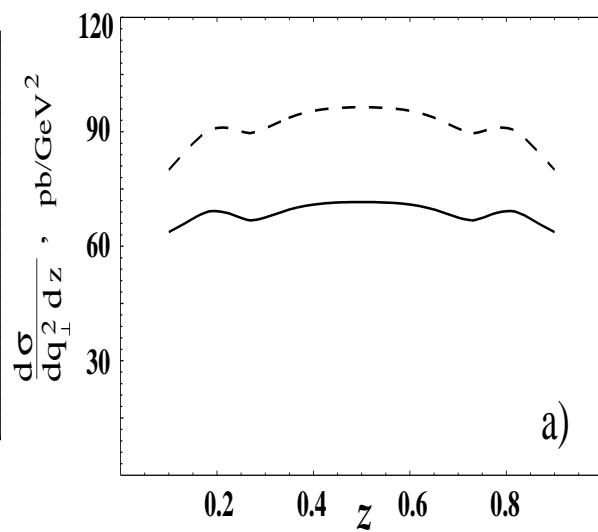
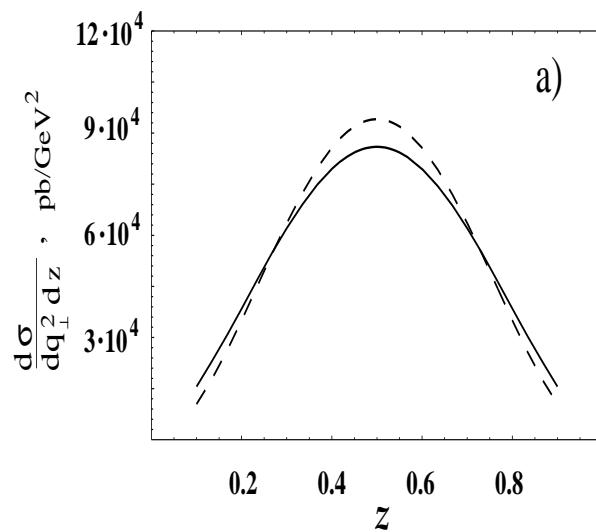
**Photoproduction: V.M. Braun *et al.* PRL 89 (2002) 172001**

$k_{\perp} = 2 \text{GeV}/c$ , chiral-odd

$\uparrow\uparrow, L_z = 0$

$k_{\perp} = 5 \text{GeV}/c$ , chiral-even

$\uparrow\downarrow, L_z = 1$



# Measurements of the photon light-cone wave function ZEUS/HERA/DESY

## Channels:

$\gamma p \rightarrow \mu^+ \mu^- p$   
 $\gamma^* p \rightarrow 2J p$   
 $\gamma^* p \rightarrow \pi^+ \pi^- p$

Janusz Szuba, UMM Cracow  
Iuliana Cohen, TAU  
Eyal Nevo, TAU

Justyna Ukleja, Warsaw U.  
Janusz Szuba, UMM Cracow  
Justyna Ukleja, Warsaw U.

## Acknowledgements:

Halina Abramowicz

Vladimir Braun, Stan Brodsky, Leonid Frankfurt, Markus Diehl, Boris Kopeliovich  
....

## Electromagnetic component of LCWF:

### Photoproduction of DiMuons (Bethe-Heitler)

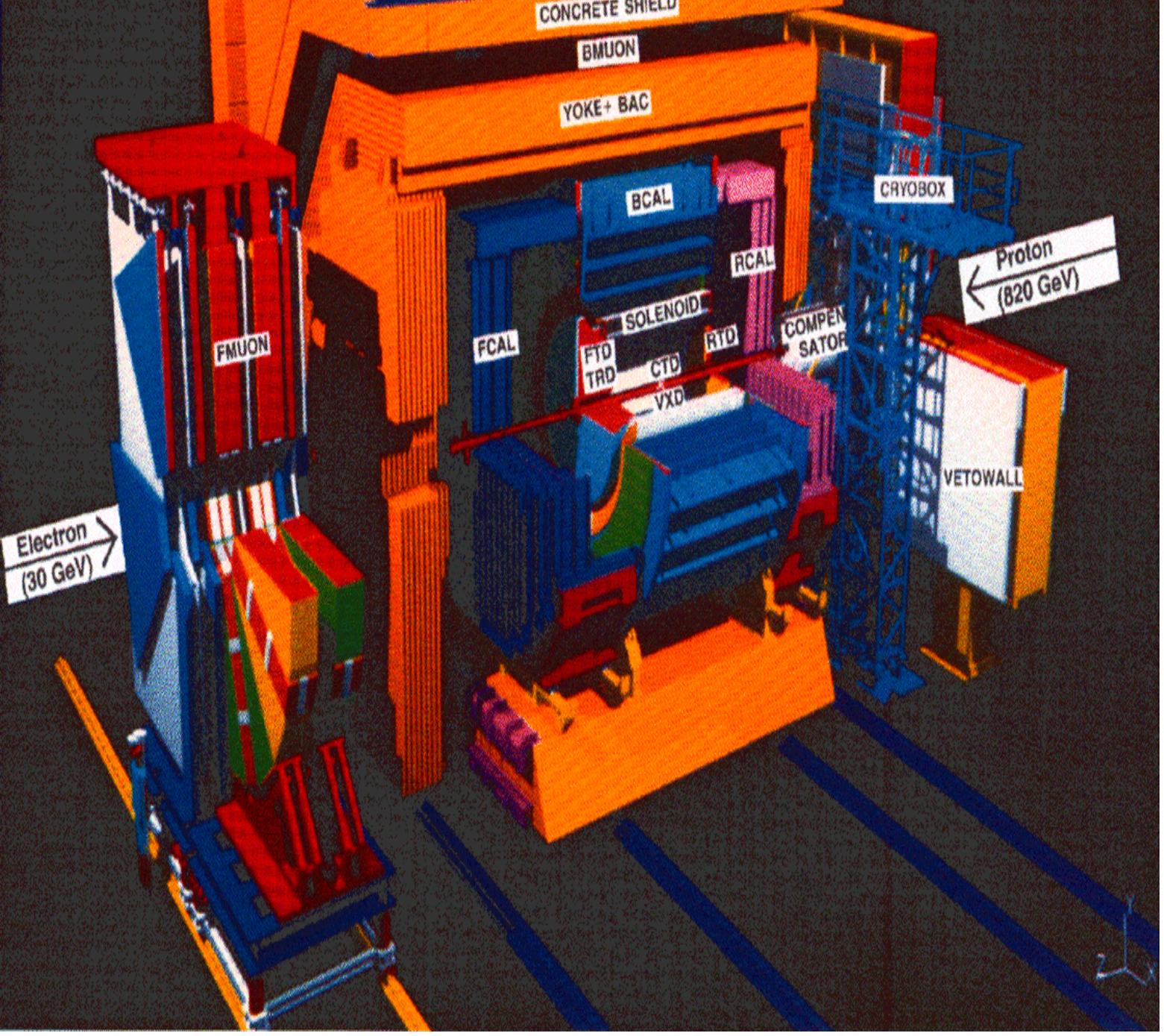
$$\frac{d\sigma_T}{dt \ du \ dk_t^2} \Big|_{t=0} \propto Tr \left| \frac{\partial}{\partial k_\mu} \psi_\mu^T \right|^2$$

$$\frac{d\sigma_T}{dt \ du \ dk_t^2} \propto \frac{4m_l^2 k_t^2 + 2(k_t^4 + a^4)[u^2 + (1-u)^2]}{(a^2 + k_t^2)^4}$$

$$a^2 = Q^2 u(1-u) + m_l^2.$$

For real photons and using  $m_l = 0 \rightarrow a = 0$ :

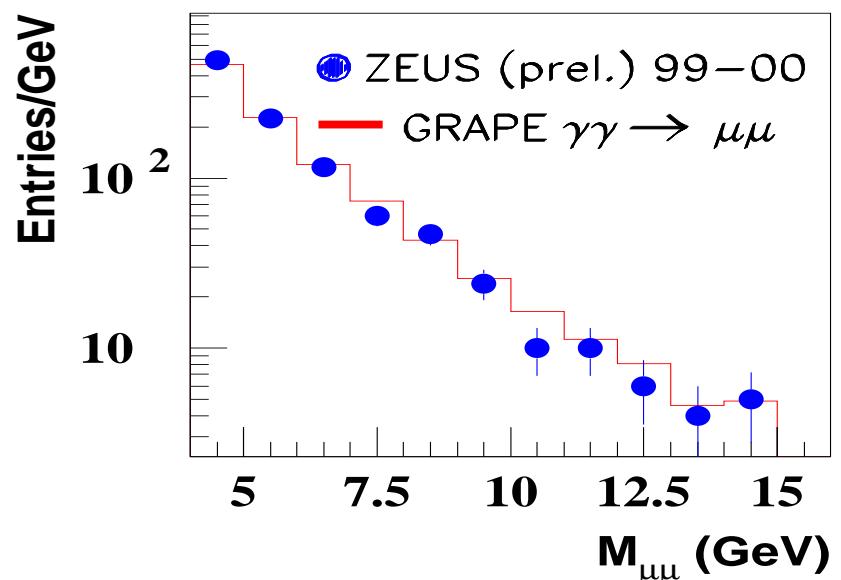
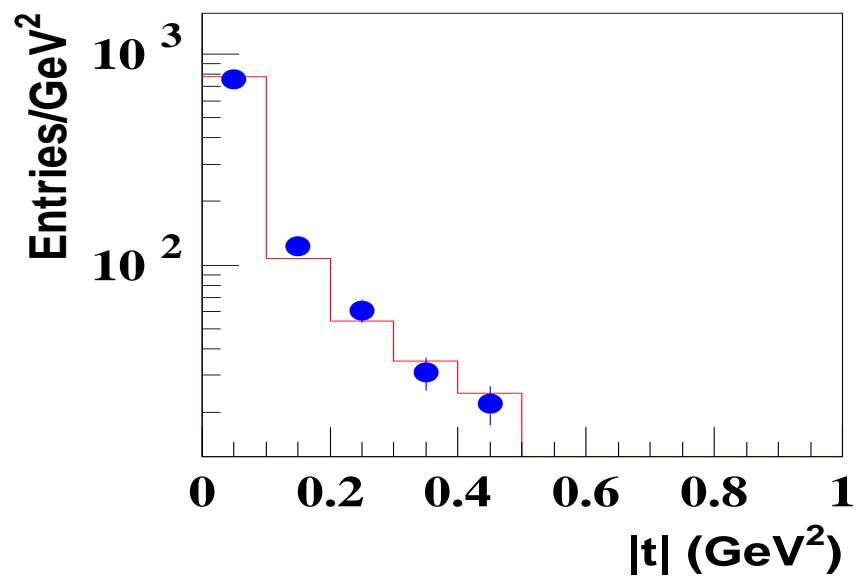
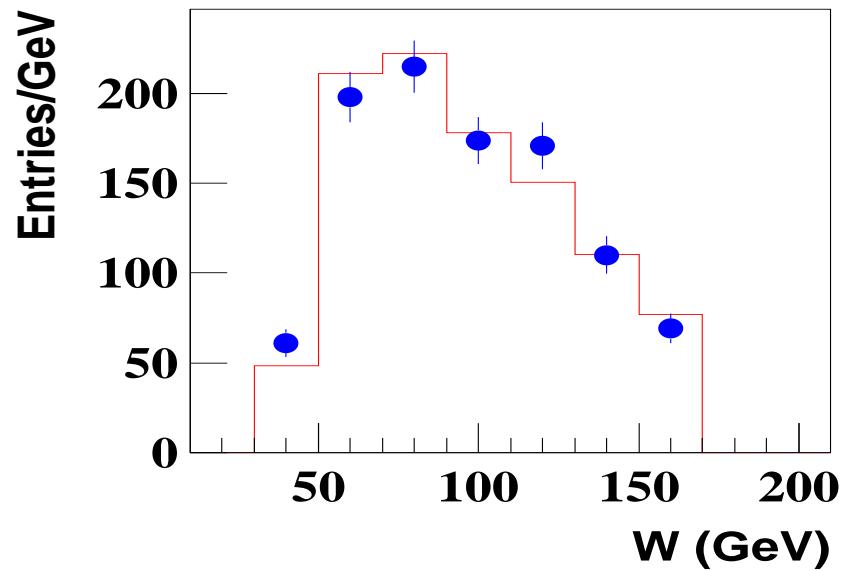
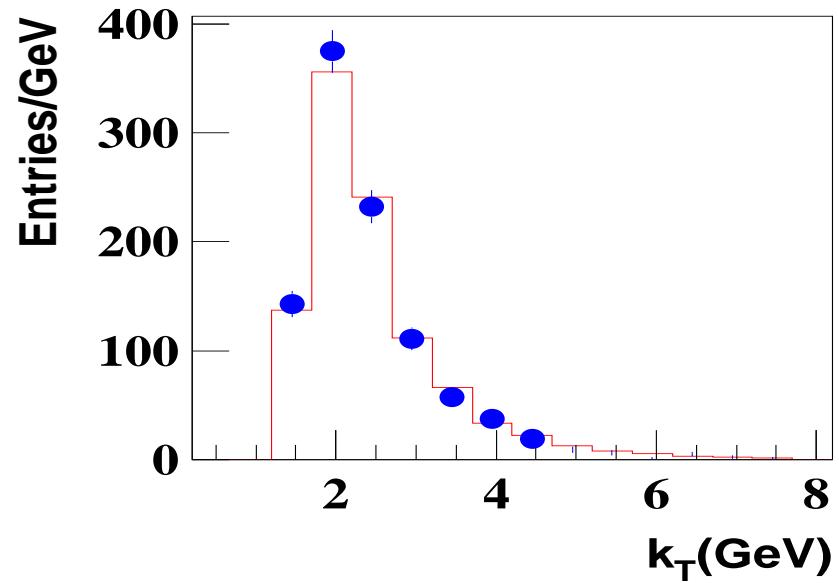
$$\frac{d\sigma_T}{dt \ du \ dk_t^2} \propto \frac{2[u^2 + (1-u)^2]}{k_t^4} \sim \frac{\Phi^2}{k_t^2}.$$



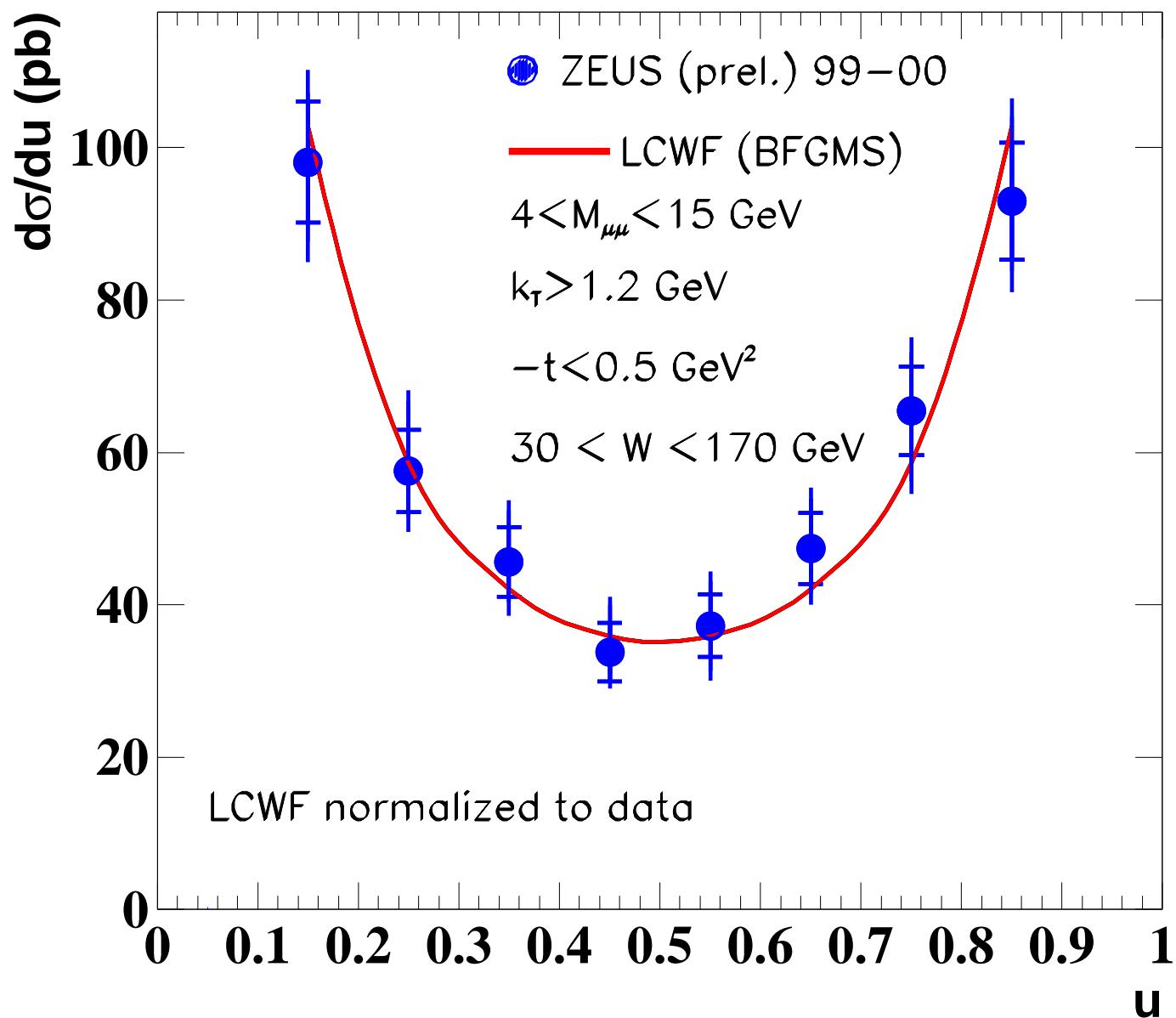
## Event Selection for $\gamma p \rightarrow \mu^+ \mu^- p$

- $\mu$  Trigger
- Proton did not disintegrate
- Elasticity (only  $2\mu$  in event)
- Diffractive (small  $t$ )
- $4 \leq M_{\mu^+ \mu^-} \leq 15 GeV$  (avoid resonances)
- Various cleaning cuts

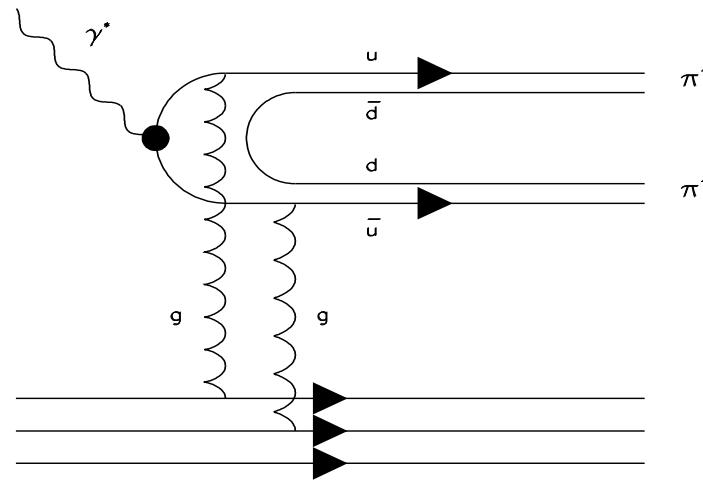
# ZEUS



# ZEUS



## Hadronic component - special case: $\gamma^* p \rightarrow \pi^+ \pi^- p$



Relation to pion Time-Like form factor ?

$$\frac{\sigma(\gamma^* + p \rightarrow 2\pi + p)}{\sigma(\gamma^* + p \rightarrow X + p)} \propto |F_\pi|^2$$

Pion quantum numbers ?      Longitudinal/Transverse ?

## Pomeron - Odderon interference

The Odderon: 3g color singlet, Charge-parity C = -1

$C(\pi^+\pi^-) = (-)^\ell$ : produced by both Pomeron and Odderon.

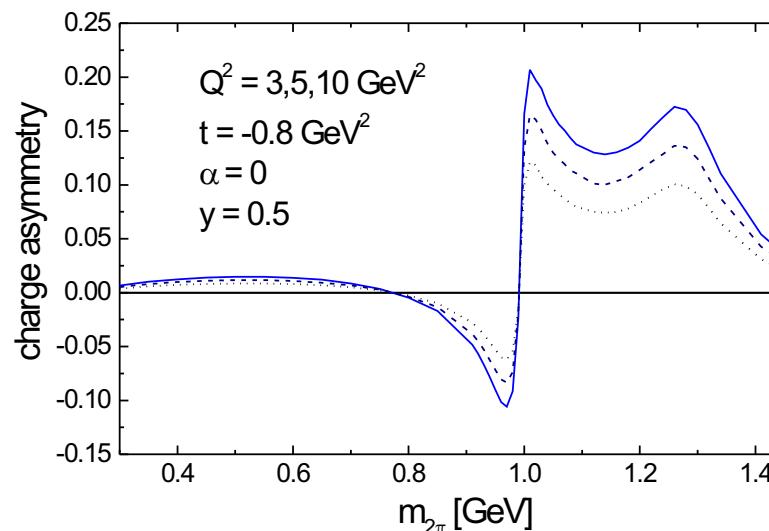
Pomeron-Odderon Interference:

Brodsky, Rathsman and Merino, Phys. Lett. B461, 114 (1999)

Results in charge asymmetry:

$$A = \frac{u(\pi^+) - u(\pi^-)}{u(\pi^+) + u(\pi^-)}$$

Hagler, Szymanowski and Teryaev Phys. Lett. B535, 117 (2002):

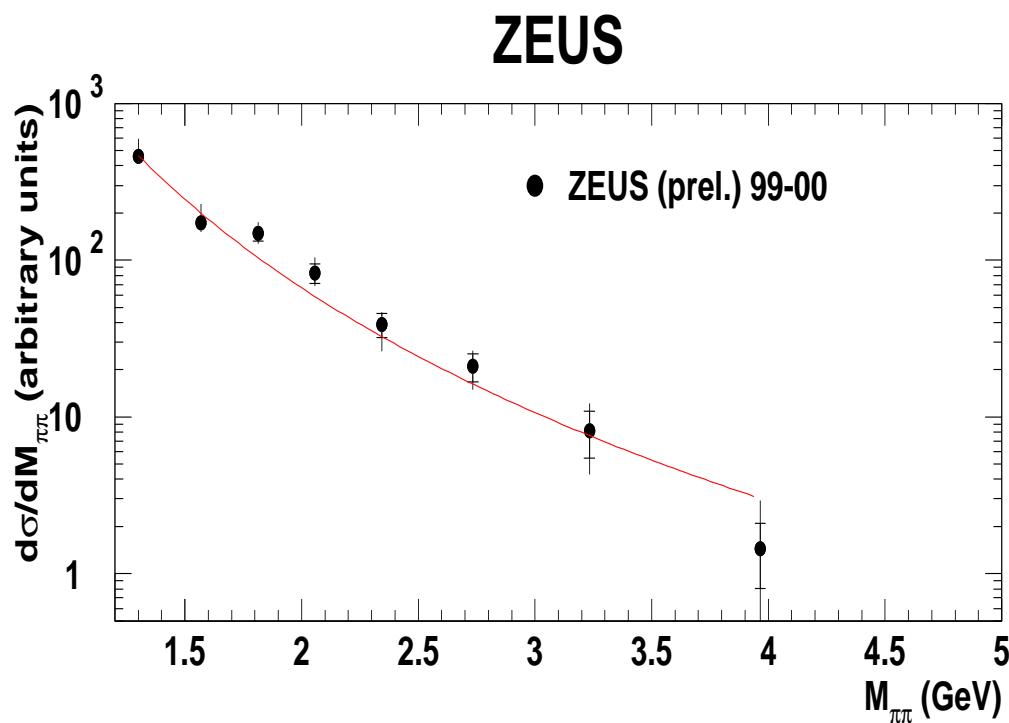


## Event Selection for $\gamma^* p \rightarrow \pi^+ \pi^- p$

- $E_{e'} > 10 \text{ GeV}$
- Proton did not disintegrate
- Pion identification by Neural Net
- Elasticity (only  $2\pi$  in event)
- Various cleaning cuts
- Diffractive (small  $t$ )
- $1.2 \leq M_{\pi^+\pi^-} \leq 5 \text{ GeV}$
- $2 < Q^2 < 20 \text{ GeV}^2$
- $40 < W < 120 \text{ GeV}$

$$\beta = \frac{Q^2}{Q^2 + M_X^2}$$

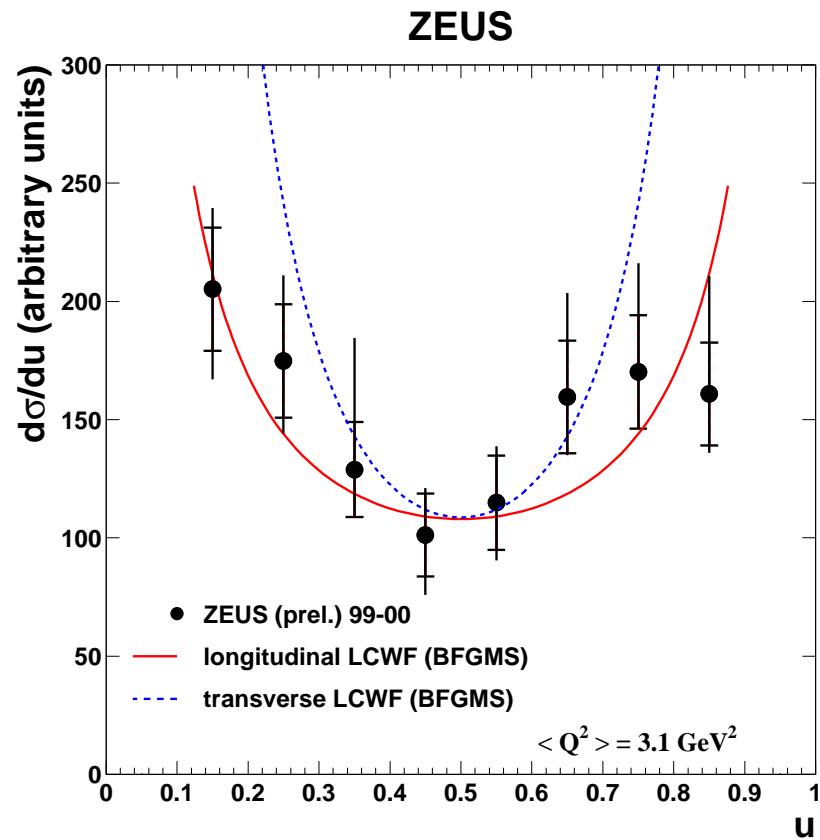
## Mass Dependence



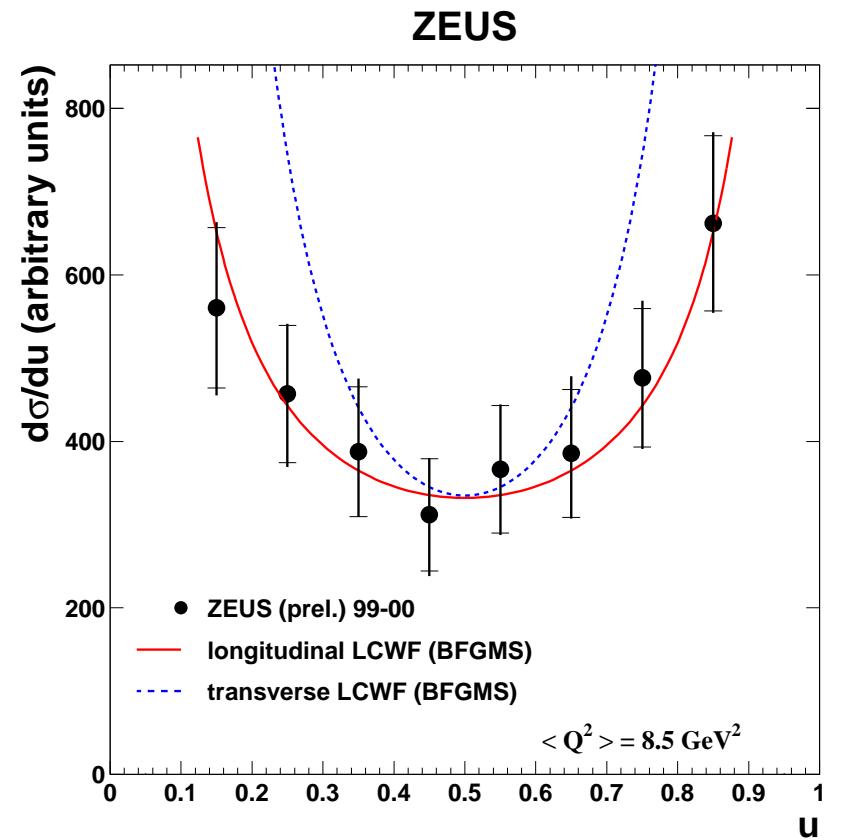
Fit :  $M_{\pi\pi}^{-4.5}$

Future derivation of Time-Like  $|F_\pi|^2$

## $u$ Dependence



$$\langle \beta \rangle = 0.52$$



$$\langle \beta \rangle = 0.75$$

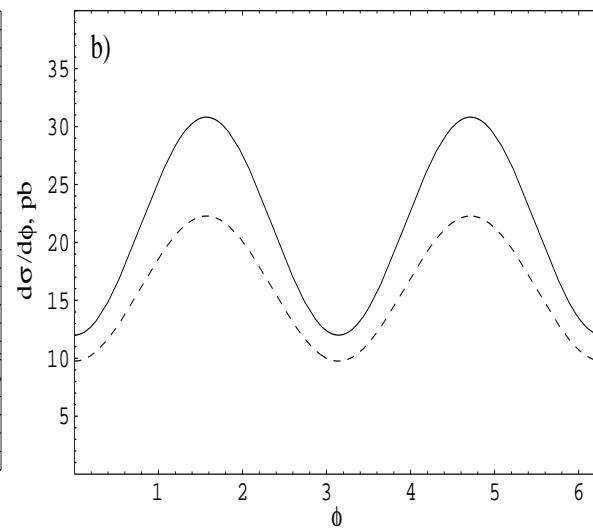
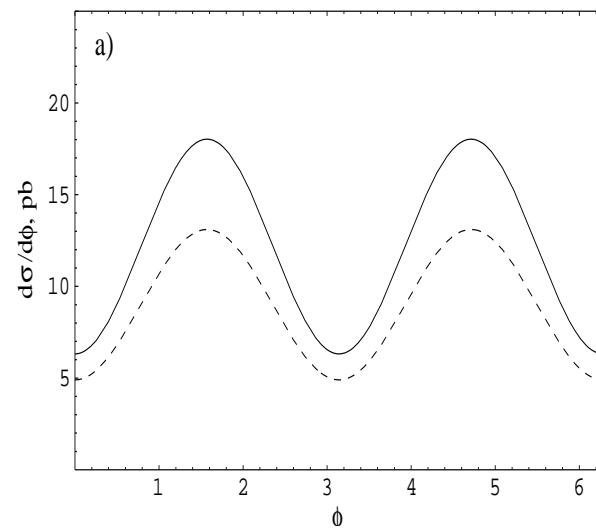
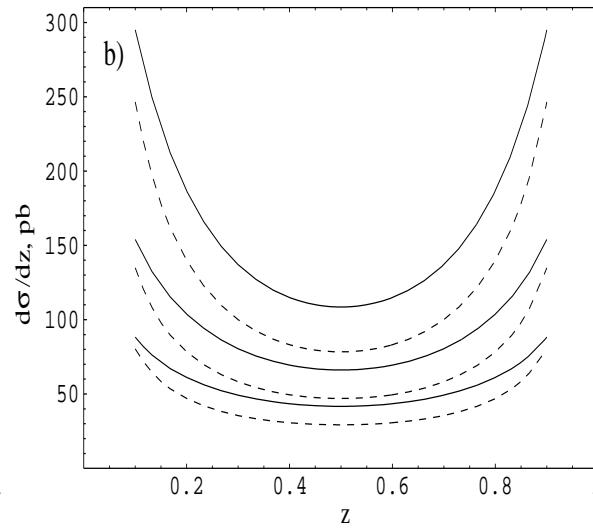
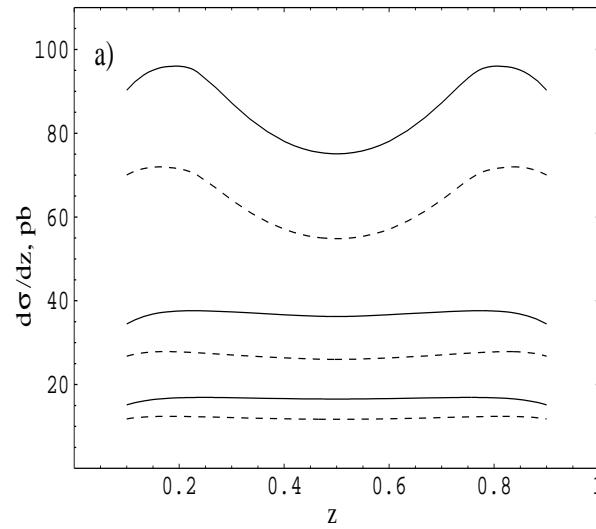
Longitudinal photon fluctuating to  $q\bar{q}$ .

**Dijet Electroproduction: V.M. Braun and D.Yu. Ivanov, Phys.Rev. D72 034016 (2005)**

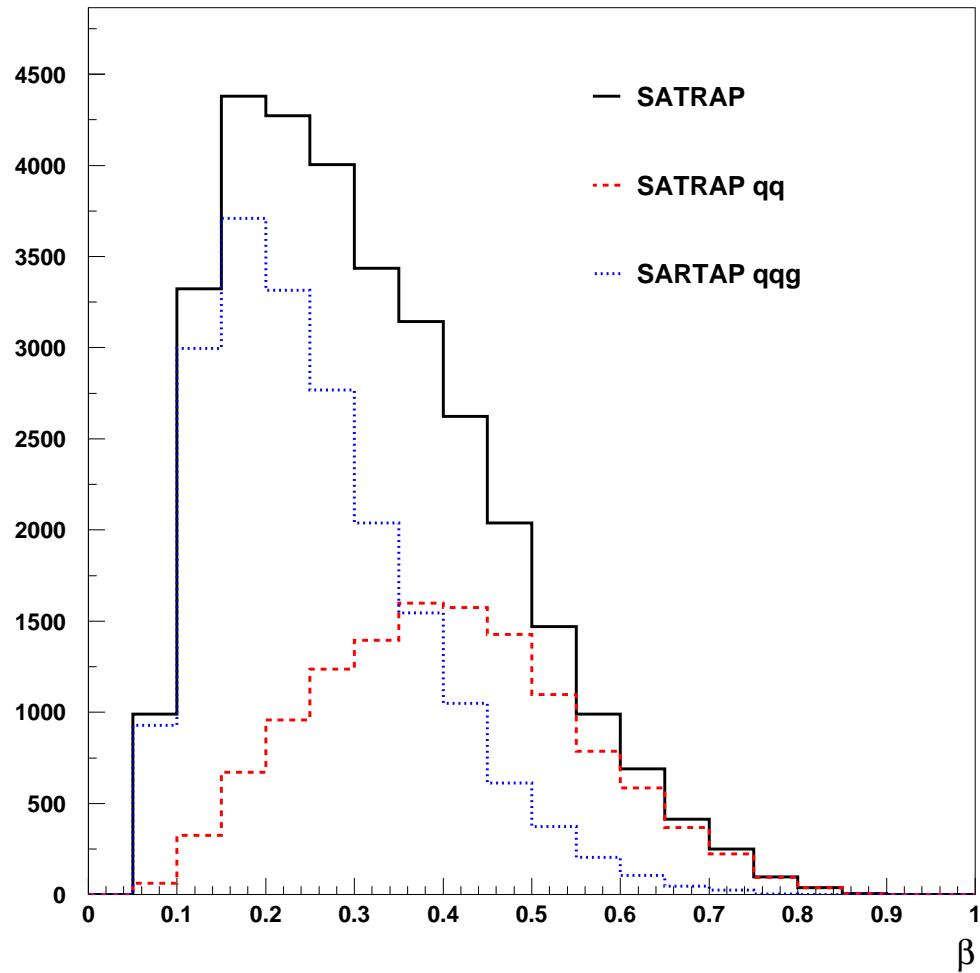
$k_{\perp}=1.25, 1.5, 1.75 \text{ GeV}/c$ ; Solid: CTEQ6L, dashed: MRST2001LO

$\beta > 0.5$

$\beta > 0$



## Selection of $q\bar{q}$ dijets using $\beta$ :



For  $\beta > 0.45$ , 23%  $q\bar{q}g$  background

## Event Selection for $\gamma^* p \rightarrow DJ$

- $E_{e'} > 10 \text{ GeV}$
- Proton did not disintegrate
- Jet identification with clustering algorithm
- Elasticity (only 2 jets in event)
- Various cleaning cuts
- Diffractive (small  $t$ )
- $5 \leq M_{DJ} \leq 30 \text{ GeV}$
- $10 < Q^2 < 500 \text{ GeV}^2$
- $100 < W < 200 \text{ GeV}$
- $0.01 < y < 0.9$
- $k_t > 1.25 \text{ GeV}$
- $\beta > 0.45$ , **Test by  $\phi$  Angular Distribution**
- $0.1 < u < 0.9$
- $0.00001 < x_{IP} < 0.015$

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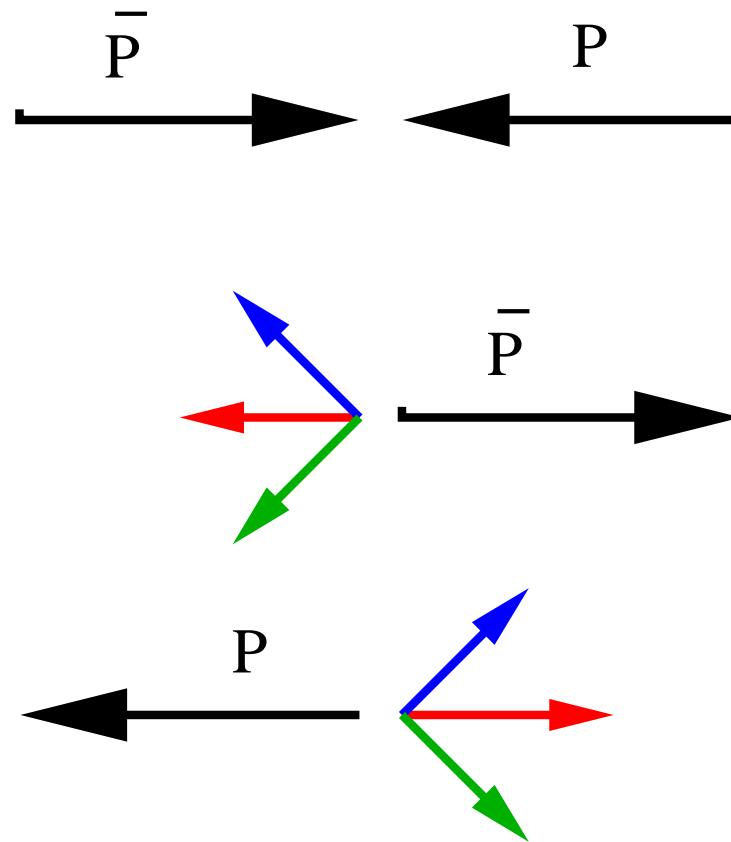
For experimental results....

stay tuned

# The Proton Light-Cone Wave Function

1. HERA:  $p e \rightarrow 3J e'$

2. Fermilab Collider:  $p \bar{p} \rightarrow 3J p$ , use Roman Pots.



## Summary

- Diffractive dissociation of hadrons and photons can be used to study their internal quark structure.
- The momentum wave function of the  $|q\bar{q}\rangle$  in the pion is described well at  $Q^2 > 10 \text{ GeV}^2$  by the Asymptotic wave function.
- For lower  $Q^2$  values the momentum wave function of the  $|q\bar{q}\rangle$  in the pion contains  $2^{nd}$  and  $4^{th}$  Gegenbauer Polynomials with coefficients:  $a_2 = 0.30 \pm 0.05$ ,  $a_4 = (0.5 \pm 0.1) \cdot 10^{-2}$ .

- Measurements of the photon electromagnetic light-cone wave functions are completed and the results are in agreement with QED.
- This provides the first proof that diffractive dissociation of particles can be reliably used to measure their light cone wave functions.

- Measurements of the photon hadronic light-cone wave functions are in progress.
- Photon dissociation  $\gamma^* p \rightarrow \pi^+ \pi^- p$  is dominated by Longitudinal photons fluctuating to  $q\bar{q}$ . Charge asymmetry as signal of the Odderon being studied.
- Photon dissociation  $\gamma^* p \rightarrow J J p$  is dominated by Transverse photons fluctuating to  $q\bar{q}$  as observed in the azimuthal angular distribution.